Multi-Name Credit Derivatives Pricing and Risk Premia

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Outline

- Overview
  - Credit Derivatives Market
  - Current Challenges
- Model Setup
  - Default Intensity Model
  - Hedging
- Time Series Analysis
- Risk Premia
  - Physical Default Dynamics
  - Types of Risk Premia
  - Decomposition of Returns
- Summary, Open Questions

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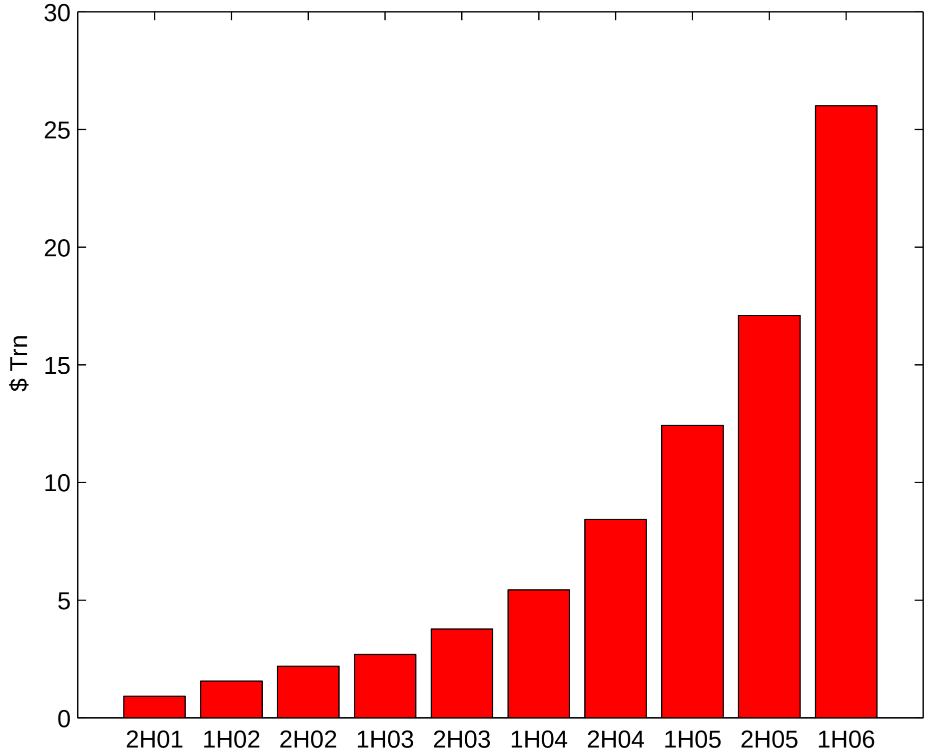
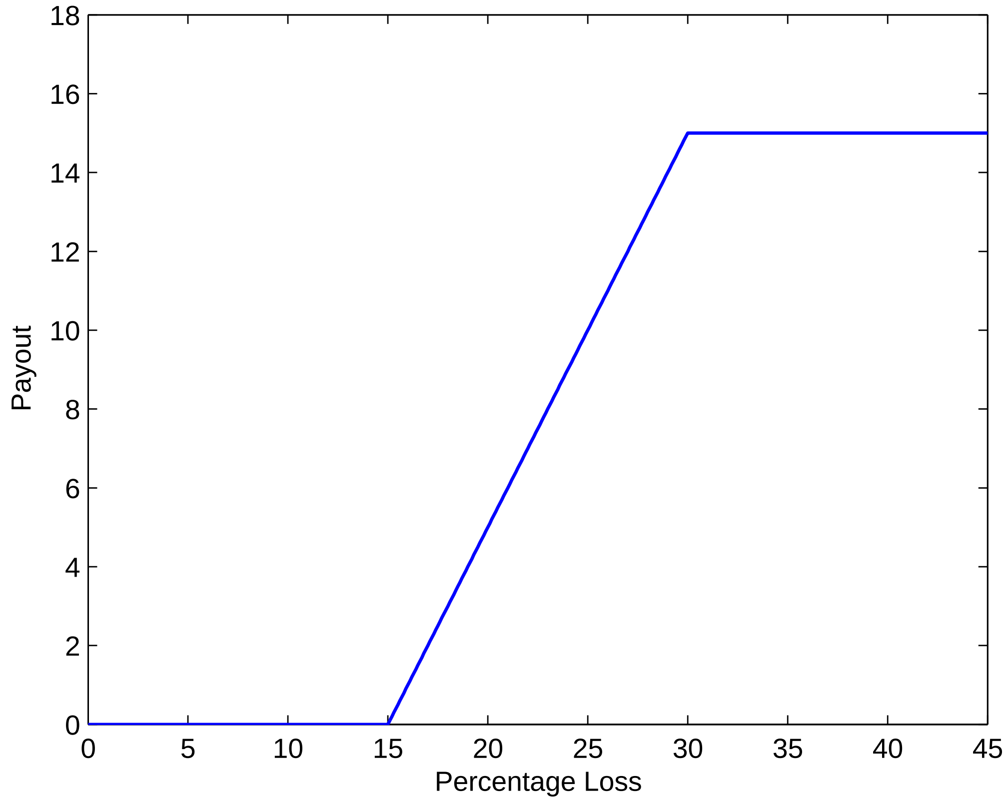
Overview - Part I

- Credit Default Swap (CDS): Protects against default of particular company
- Notional amount of outstanding credit default swaps:



Overview - Part II

- Credit Indices: Protects against default of basket of companies
- Credit Index Tranches



Overview - Part II

- CDX.NA.IG index:
  - Underlying portfolio: 125 North-American investment-grade issuers
  - Tranche structure and market prices on December 5, 2005:

Tranche	% of Credit Losses	Spread (bps)	Up-front Payment
Equity	0% - 3%	500	40.7%
Junior Mezzanine	3% - 7%	111.9	0
Mezzanine	7% - 10%	31.3	0
Senior	10% - 15%	13.5	0
Super Senior	15% - 30%	7.4	0
Index	0%- 100%	49	0

Overview - Part IV

- Current Challenges
  - No ”Black-Scholes” model yet
  - Defaults are rare events $\Rightarrow$ default correlations hard to estimate
  - Intensity based models look quite promising, although computationally burdensome
- Recent work in this area: Duffie and Singleton (1997), Lando (1998), Duffie and Gârleanu (2001), Giesecke and Goldberg (2005), Mortensen (2006), Feldhütter (2007)
- Today:
  - Stochastic intensity model that is computationally quite tractable: 3-5x speed up
  - Joint model for physical ($\lambda_i^{\mathbb{P}}$) and risk-neutral ($\lambda_i^{\mathbb{Q}}$) default intensities $\Rightarrow$ Risk premia

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Default Time Model - Part I

- Building blocks for default intensity model: Basic Affine Jump Diffusions (AJD)

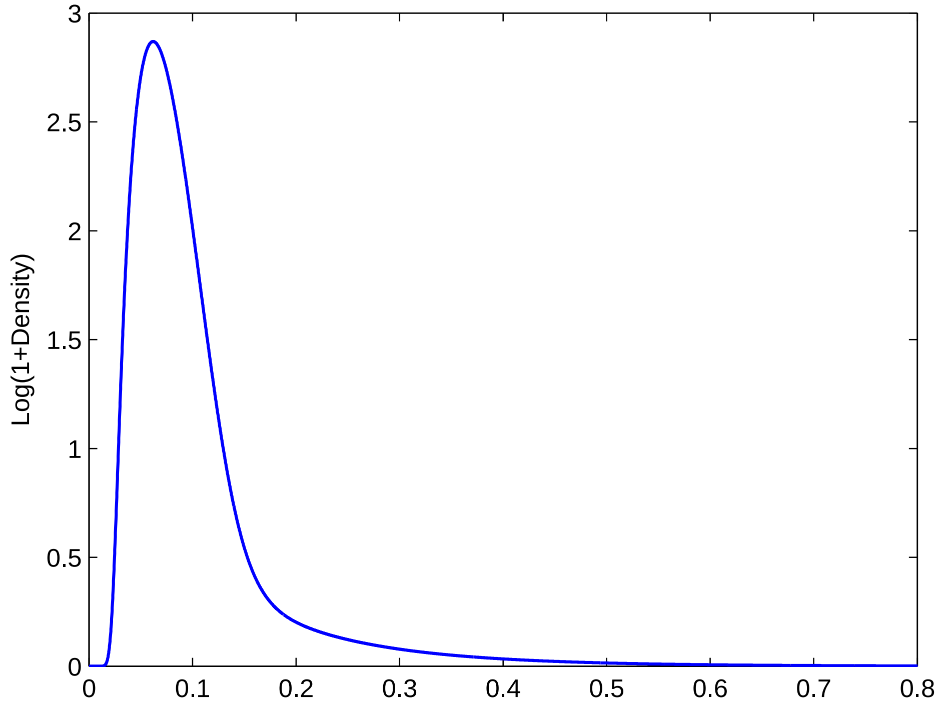
$$dZ_t = \kappa^{\mathbb{Q}} \left(\theta^{\mathbb{Q}} - Z_t\right) dt + \sigma \sqrt{Z_t} dB_t^{\mathbb{Q}} + dJ_t^{\mathbb{Q}}$$

  - $B_t^{\mathbb{Q}}$ Brownian motion under $\mathbb{Q}$
  - $J_t^{\mathbb{Q}}$ an independent compound Poisson process with jump intensity $l^{\mathbb{Q}}$ exponentially distributed jumps with mean $\mu^{\mathbb{Q}}$

- Following quantities known explicitly (Duffie, Pan, and Singleton (2000)):
  - Moment generating function: $E^{\mathbb{Q}}\left(e^{q\int_0^T Z_t^{\mathbb{Q}} dt}\right)$
  - Fourier transform: $E^{\mathbb{Q}}\left(e^{iq\int_0^T Z_t^{\mathbb{Q}} dt}\right)$

Default Time Model - Part II

- The density of $\int_0^T Z_t dt$ can obtained by Fourier inversion (e.g. via FFT)
- Example: $Z_0 = 0.01$, $k = 0.25$, $\theta = 0.02$, $\sigma = 0.05$, $l = 0.02$, $\mu = 0.08$, $T = 5$



Default Time Model - Part III

- Factor model for default intensities

$$\lambda_{it}^{\mathbb{Q}} = X_{it} + a_i Y_t, \tag{1}$$

as in Duffie and Gârleanu (2001), and Mortensen (2006), where X_i and Y are independent basic AJD

- Conditional on $\{\lambda_{it}^{\mathbb{Q}} : t \geq 0\}$, τ_i is the time of the first jump of an inhomogeneous Poisson process with intensity $\lambda_i^{\mathbb{Q}}$

- Survival Probabilities

$$\mathbb{Q}\left(\tau_i > t\right) = E^{\mathbb{Q}}\left[e^{-\int_0^t \lambda_{i,s}^{\mathbb{Q}} ds}\right] = E^{\mathbb{Q}}\left[e^{-\int_0^t X_{i,s} ds}\right] E^{\mathbb{Q}}\left[e^{-a_i \int_0^t Y_s ds}\right]$$

Default Time Model - Part IV

- Conditional on $\widetilde{Y}_t := \int_0^t Y_s ds$, defaults in $(0, t]$ are independent and default probabilities given by

$$\mathbb{Q}\left(\tau_i \leq t \mid \widetilde{Y}_t\right) = 1 - E^{\mathbb{Q}}\left[e^{-\int_0^t X_{i,s} ds}\right] e^{-a_i \widetilde{Y}_t}$$

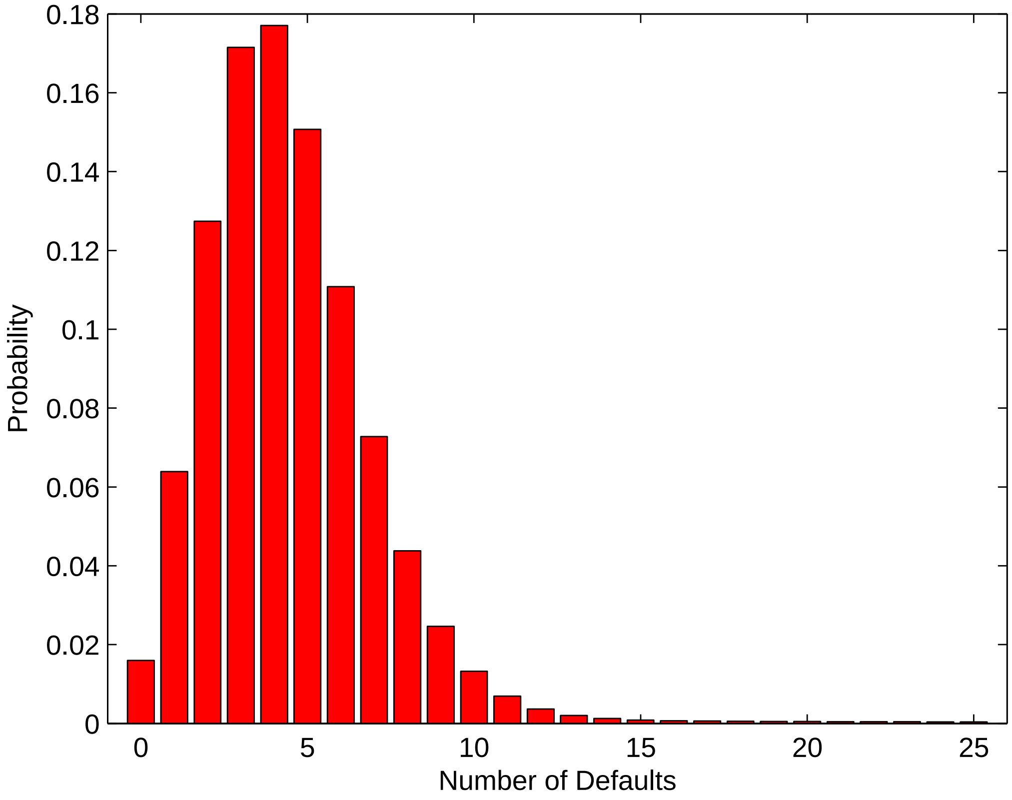
- The conditional distribution of number of defaults $P_t(k \mid \widetilde{Y}_t)$ can therefore be obtained in a simple recursive manner (Andersen, Sidenius, and Basu (2003)): Convolution of Bernoulli R.V.s (default indicator variables)

- Unconditional distribution of number of defaults

$$P_t\left(k\right) = \int P_t(k \mid \widetilde{Y}_t) d\mathbb{Q}\left(\widetilde{Y}_t\right)$$

Default Time Model - Part V

- Distribution of number of defaults for $T = 5$, implied by the model fitted to tranche spreads on December 5, 2005



Pricing

- Model-implied spread for CDS, credit tranches, credit index:

  Value of Protection Leg = Value of Fixed Leg = $PV_{01} \times$ Spread

- Assuming (under $\mathbb{Q}$)
  - Default intensities and interest rates independent
  - Recovery rates independent of default intensities
  - Defaults occur on average in middle between two coupon payment dates

  $\Rightarrow$ model-implied CDS, tranche and index spreads are an explicit function of the portfolio loss distribution $P_{t_k}(k)$ at all future coupon payment dates t_k

  $\Rightarrow$ By calculating $P_t(k)$ for a small number of points in time t, we can price large class of credit derivative securities

Computational Tricks

- Spline interpolation of Fourier transform
- Restrict ASB-algorithm to values of k, such that $P_t(k \mid \widetilde{Y}_t) > 10^{-10}$
- Gauss-Legendre integration for calculating unconditional portfolio loss distribution $P_t\,(k)$
- Geometric interpolation of portfolio loss distribution $P_t(k)$ over t

$\Rightarrow$ For fixed set of parameters, pricing of tranches in 1-2 seconds

Recovery Rates - Part I

- Recovery Rate = Market value of the underlying debt as a fraction of the notional amount at the time of default
- Average recovery rate for senior unsecured bonds 1970-1998: about 40%
- Well documented empirical features (Moody's (2000), Altman, Bray, Resti, and Sironi (2003)):
  - Randomness: 25th and 75th percentile 30% and 65%, respectively
  - Serial Correlation
  - Counter-cyclical recovery rates
- Usually, assumption of constant recovery rates only innocuous in univariate setting, where <u>expected</u> losses matter

Recovery Rates - Part II

- Sufficient for pricing, knowledge of:

$$\mathbb{Q}_t \left(L_T \leq x \right) \qquad \forall T > t, x \in \mathbb{R}_+$$

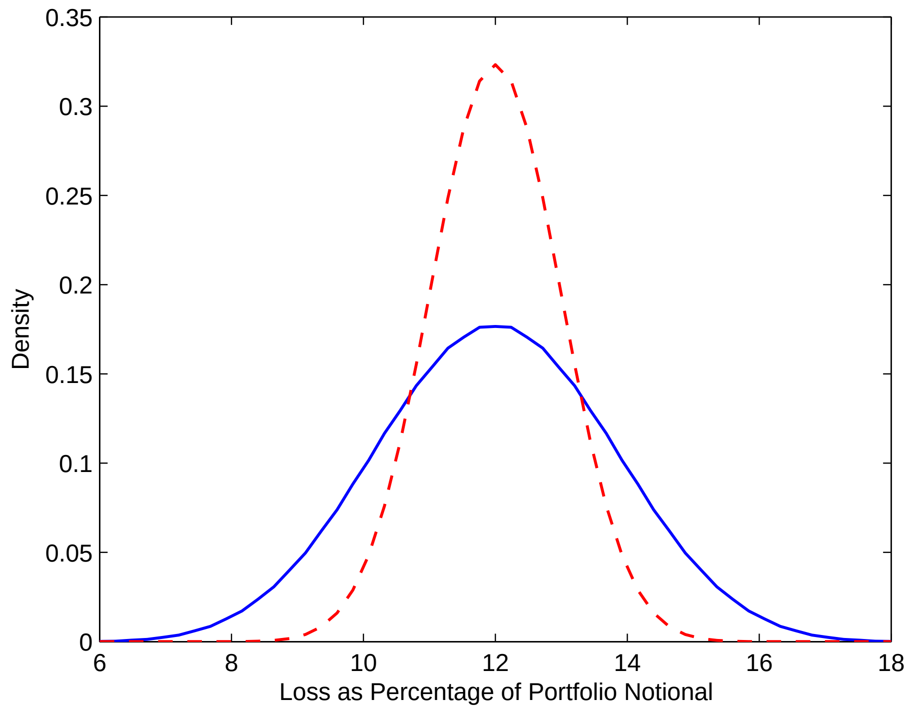
- Rewrite:

$$E_t^{\mathbb{Q}} \left(\mathbb{Q} \left(L_T \leq x \,\middle|\, \text{n Defaults} \right) \right) = \sum_{n=0}^{m} P_{t,T}(n) G_n \left(x \right)$$

  where G_n is the portfolio loss distribution conditional on seeing n defaults, assumed to be independent of T

- Different choices for G_n:
  - Constant RRs equal to 40%: $G_n(x) = \mathbf{1}_{\{x \geq 0.6 * n/m\}}$
  - Stochastic, but uncorrelated RRs:
    $G_1 = \frac{1}{m} \left(1 - \text{Uniform} \left(\{0.1, 0.4, 0.7\} \right) \right)$, $G_n = G_1 * G_{n-1}$ for $n \geq 2$

- Stochastic and serially correlated RRs: Recovery of n-th default modeled as (time-inhomogeneous) Markov chain with state space $\{0.1, 0.4, 0.7\}$, representing a bad, medium, good economic environment
- G_{25} for stochastic but independent recovery rates (dashed line), for stochastic and serially correlated recovery rates (solid line, 80% probability of staying in same state):



Model Estimation - Part I

- Assume market quotes for CDS and tranche spreads subject to normally distributed measurement noise, for example:

$$cp^*_{t,j,M} = cp_{t,j,M} + \varepsilon^{tr}_{t,j,M}$$
$$\varepsilon^{tr}_{t,j,M} \sim N(0, \sigma^2_{tr}(cp^*_{t,j,M})^2)$$

- Likelihood function of form

$$\log LH_{tr}\left(\Theta^{\mathbb{Q}}_{tr}\right) = c_{tr} + d_{tr} \cdot RMSE^2_{tr},$$

for constants c_{tr} and $d_{tr} < 0$ and

$$RMSE_{tr} = \sqrt{\frac{1}{T}\frac{1}{J}\frac{1}{M}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{M\in\{5,7,10\}}\left(\frac{cp_{t,j,M} - cp^*_{t,j,M}}{cp^*_{t,j,M}}\right)^2} \quad (2)$$

Model Estimation - Part II

- To ensure model identifiability impose:

$$\frac{1}{m}\sum_{i=1}^{m} a_i = 1$$

- To get parsimonious model impose:

$$\kappa_i^{\mathbb{Q}} = \kappa_Y^{\mathbb{Q}} =: \kappa^{\mathbb{Q}}, \quad \sigma_i = \sqrt{a_i}\sigma_Y =: \sqrt{a_i}\sigma, \quad \mu_i^{\mathbb{Q}} = a_i \mu_Y^{\mathbb{Q}},$$

$$\omega_1 = \frac{l_Y}{l_i + l_Y}, \qquad 1 \le i \le m$$

and

$$\omega_2 = \frac{a_i \theta_Y}{a_i \theta_Y + \theta_i}, \qquad 1 \le i \le m$$

- Ensure that $\lambda_{it}^{\mathbb{Q}} \overset{\mathbb{Q}}{\sim} AJD(\lambda_{i,0}^{\mathbb{Q}}, \kappa^{\mathbb{Q}}, \theta_i^{\mathbb{Q}} + a_i \theta_Y^{\mathbb{Q}}, \sqrt{a_i}\sigma_Y, l_i^{\mathbb{Q}} + l_Y^{\mathbb{Q}}, \mu_i^{\mathbb{Q}})$

Model - Part III

- For fixed $\kappa^{\mathbb{Q}}, \theta_Y^{\mathbb{Q}} + Avg(\theta_i^{\mathbb{Q}}), \sigma_Y, l_Y^{\mathbb{Q}} + l_i^{\mathbb{Q}}, \mu^{\mathbb{Q}}, \omega_1, \omega_2$
  - Calibrate term-structure of CDS quotes by varying a_i, $\lambda_{i,0}^{\mathbb{Q}}$
  - Calculate model-implied tranche spreads
  - Calculate relative RMSE (2)
- By varying the parameters $\kappa^{\mathbb{Q}}, \theta_Y^{\mathbb{Q}} + Avg(\theta_i^{\mathbb{Q}}), \sigma_Y, l_Y^{\mathbb{Q}} + l_i^{\mathbb{Q}}, \mu^{\mathbb{Q}}, \omega_1, \omega_2$, minimize relative RMSE given by (2), for example using Nelder-Mead Simplex method

Results: Model Fit - Part I

- Comparison of model fit to 5-year tranche spreads on December 5, 2005:

Tranche	Bloomberg	Model$_M$	Markit	Model$_E$	Model$_{E+}$	Model$_{E++}$
0% - 3%	41.1%	43.2%	40.7%	40.5%	40.3%	40.6%
3% - 7%	117.5	125.9	111.9	118.5	123.7	121.2
7% - 10%	32.9	30.6	31.3	29.2	28.9	30.6
10% - 15%	15.8	21.3	13.5	14.6	14.5	14.4
15% - 30%	7.0	8.8	7.4	7.2	7.1	7.3
Rel. RMSE	-	0.200	-	0.056	0.072	0.049

Model$_M$ . . . Model by Mortensen (2006)

Model$_E$ . . . constant recovery rates

Model$_{E+}$ . . . stochastic but uncorrelated recovery rates

Model$_{E++}$ . . . stochastic and serially correlated recovery rates

Results: Model Fit - Part II

- Model fit on December 5, 2005 to the term-structure of tranche spreads:

Tranche	Market 5yr	Model 5yr	Market 7yr	Model 7yr	Market 10yr	Model 10yr
0% - 3%	40.7%	39.9%	54.8%	56.3%	61%	63%
3% - 7%	111.9	124.8	270.5	303.3	647	664
7% - 10%	31.3	30.3	53.5	57.7	129	122
10% - 15%	13.5	15.5	29.8	29.0	65	45
15% - 30%	7.4	7.2	11.6	12.4	23	19
Index	49	49	58	58	71	68

Results: Model Parameters

Comparison of the MLE model parameters for the fit to market prices on December 5, 2005:

	$k^{\mathbb{Q}}$	$\theta_Y^{\mathbb{Q}} + Avg(\theta_i^{\mathbb{Q}})$	$\sigma_Y^{\mathbb{Q}}$	$l_Y^{\mathbb{Q}} + l_i^{\mathbb{Q}}$	$\mu^{\mathbb{Q}}$	ω_1	ω_2	ω_3
Model$_E$	0.010	0.077	0.087	0.008	0.223	0.35	0.09	0.014
Model$_{E+}$	0.010	0.063	0.084	0.008	0.224	0.34	0.09	0.014
Model$_{E++}$	0.010	0.077	0.087	0.008	0.223	0.34	0.09	0.014

Model$_E$ . . . constant recovery rates

Model$_{E+}$ . . . stochastic but uncorrelated recovery rates

Model$_{E++}$ . . . stochastic and serially correlated recovery rates

Applications: Hedging - Part I

- Notation:
  - $CDS_i\left(t, T, \Theta^{\mathbb{Q}}\right)$ ... model-implied CDS spread for i-th company
  - $Idx\left(t, T, \Theta^{\mathbb{Q}}\right)$ ... model-implied index spread
  - $Tr_j\left(t, T, \Theta^{\mathbb{Q}}\right)$ ... model-implied spread for j-th tranche
- Calculate price sensitivities by scaling default intensities: $\lambda_{it}^{\mathbb{Q}} \leftarrow \lambda_{it}^{\mathbb{Q}}(1+\varepsilon)$
- Tranche delta with respect to index:

$$\Delta_j^{idx}(t) = \frac{\partial Tr_j\left(t, T, \Theta^{\mathbb{Q}}\right)}{\partial \varepsilon}\Big|_{\varepsilon=0} \Bigg/ \frac{\partial Idx\left(t, T, \Theta^{\mathbb{Q}}\right)}{\partial \varepsilon}\Big|_{\varepsilon=0},$$

- Hedging ratio: $HR_j^{(idx)}(t) = \Delta_j^{(idx)}(t) \times \frac{\text{Tranche Notional}}{\text{Index Notional}} \times \frac{\text{Tranche PV01}}{\text{Index PV01}}$

Applications: Hedging - Part II

- Tranche position with $\$1$ notional, and index position with $-\$HR_j\left(t\right)$ notional, eliminates exposure to market-wide changes in credit spreads (up to first-order)
- Deltas $\Delta_j^{idx}\left(t\right)$ for the 5-year CDX.NA.IG on December 5, 2005:

	0%-3%	3%-7%	7%-10%	10%-15%	15%-30%
$\Delta_{j,Copula}$	18.5	5.5	1.5	0.8	0.4
$\Delta_{j,AJD}$	21.1	5.8	1.2	0.4	0.2

- Similar: tranche deltas with respect to individual CDS

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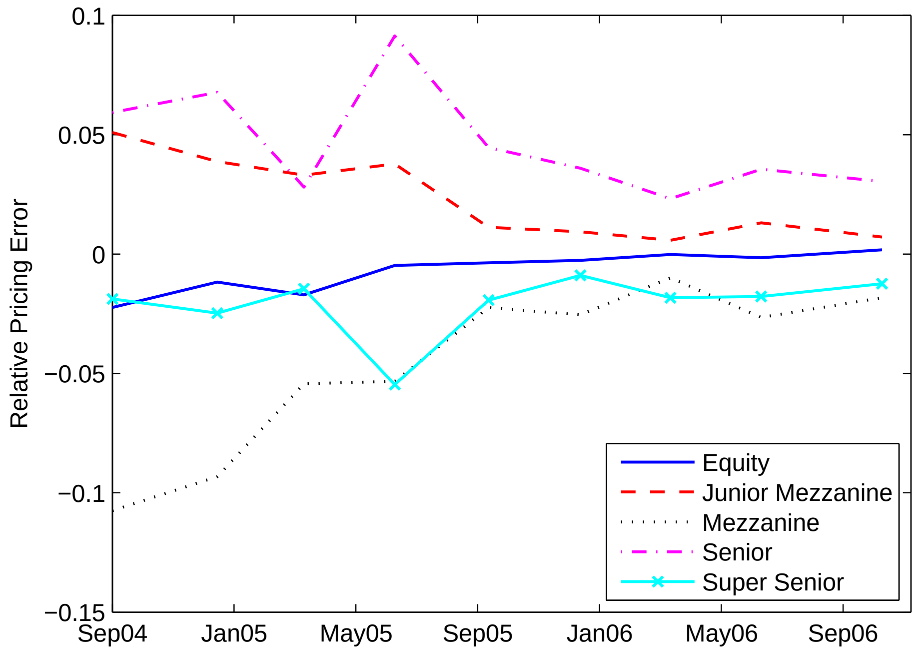
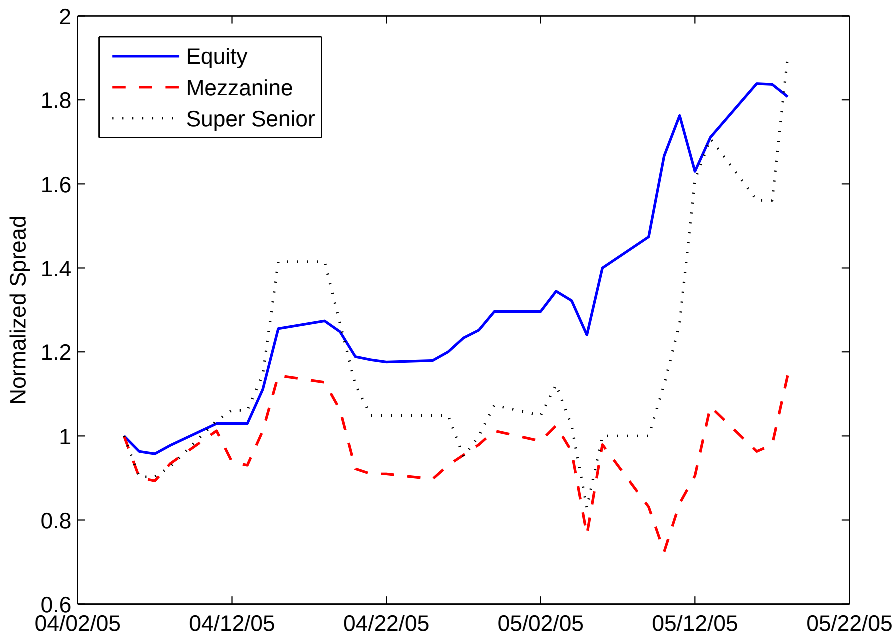
Time Series Fit - Part I

- Fixed parameters for risk-neutral default intensity dynamics:
  - Very poor fit
  - Expected, since investors's risk aversion changes over time
- Time-varying parameters:

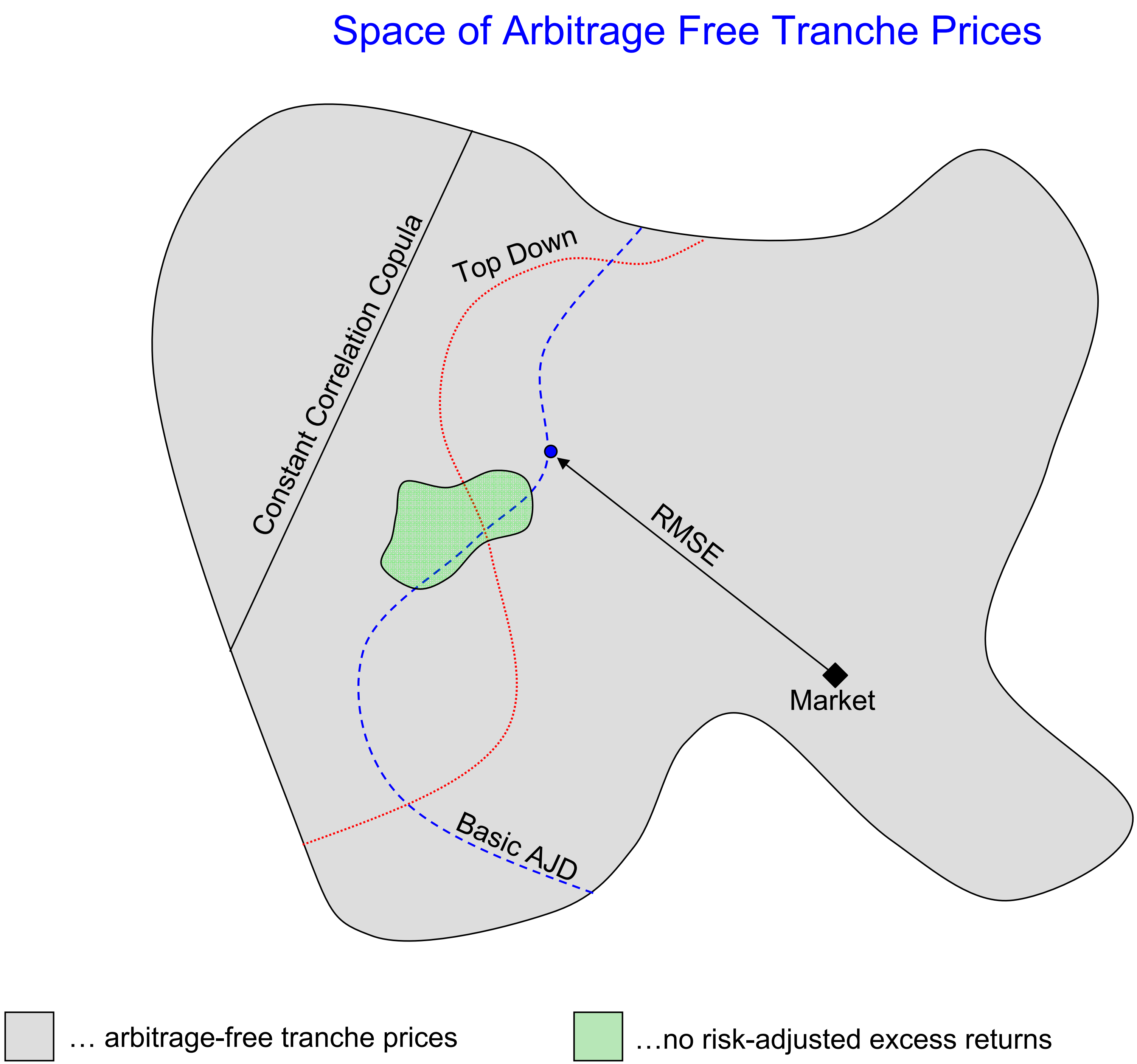


Time Series Fit - Part II

- Patterns in the tranche pricing errors:
  - General downward trend
  - Spike in May/June 2005 (”correlation crunch”)



⇒ Relative tranche pricing error proxy for market efficiency?


Space of Arbitrage Free Tranche Prices
Constant Correlation Copula
Top Down
RMSE
Market
Basic AJD
… arbitrage-free tranche prices
…no risk-adjusted excess returns

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Joint model for $\lambda_i^{\mathbb{P}}$ and $\lambda_i^{\mathbb{Q}}$ - Part I

- Ultimate goal: pin down differences between physical $(\mathbb{P})$ and risk-neutral $(\mathbb{Q})$ probability measure
- Joint model for $\lambda_i^{\mathbb{P}}$ and $\lambda_i^{\mathbb{Q}}$:

$$\begin{aligned}\lambda_{it}^{\mathbb{Q}} &= \quad X_{it} + \quad a_i^{\mathbb{Q}} Y_t, \\ \lambda_{it}^{\mathbb{P}} &= b_{it} X_{it} + \quad a_i^{\mathbb{P}} Y_t,\end{aligned}$$

  where X_i and Y are basic affine jump diffusions

- Dynamics of X_i and Y differ under $\mathbb{P}$ and $\mathbb{Q}$, for example:

$$dY_t = \kappa_Y^{\mathbb{Q}} \left(\theta_Y^{\mathbb{Q}} - Y_t\right) dt + \sigma_Y \sqrt{Y_t} dB_t^{\mathbb{Q},(Y)} + dJ_t^{\mathbb{Q},(Y)}$$

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Joint model for $\lambda_i^{\mathbb{P}}$ and $\lambda_i^{\mathbb{Q}}$ - Part II

- $\mathbb{Q}-$dynamics of $\lambda_i^{\mathbb{Q}}$ implied by market observed tranche and CDS spreads
- $\mathbb{P}-$dynamics of $\lambda_i^{\mathbb{P}}$ fitted to 25 years of corporate default data on 2,793 publicly traded companies:
  - Duffie, Eckner, Horel, and Saita (2006) estimated proportional hazard model

    $$\lambda_{it}^{\mathbb{P}} = e^{\beta \cdot W_{it}} \, e^{\eta Y_t}$$

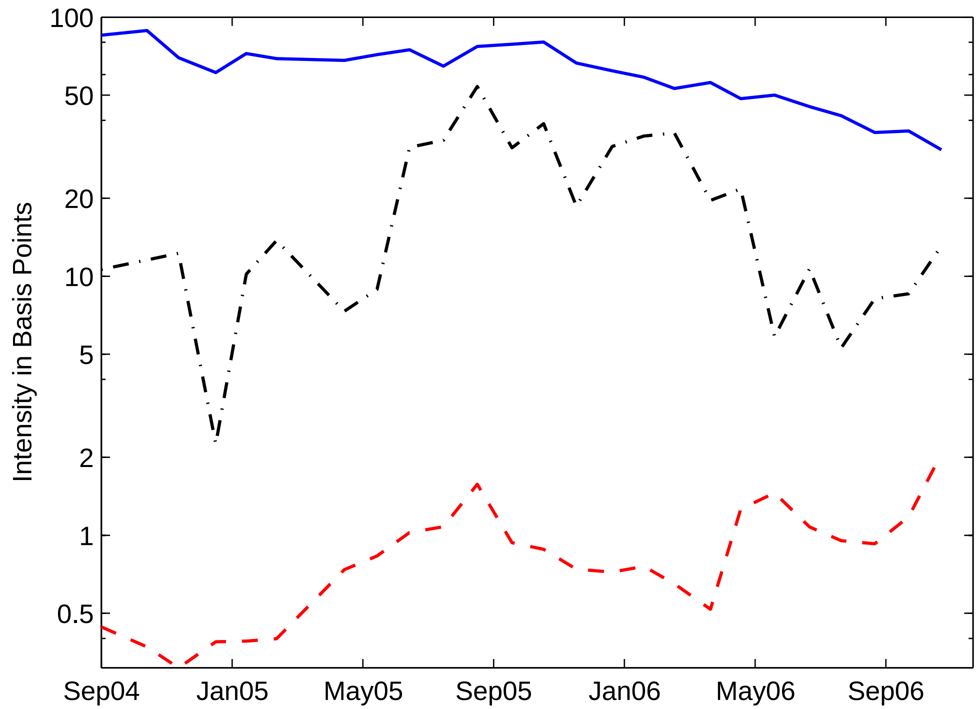
    with time-varying covariate vector W_{it} and frailty variable Y following an Ornstein-Uhlenbeck process

- Radon-Nikodym Derivative (under technical conditions):

$$E\left(\frac{d\mathbb{Q}}{d\mathbb{P}} \,\middle|\, \mathcal{F}_t\right) = \left(\prod_{j=1}^{m+1} \Lambda_{jt}^{(1)}\right) \left(\prod_{j=1}^{m+1} \Lambda_{jt}^{(2)}\right) \left(\prod_{i=1}^{m} \Lambda_{it}^{(3)}\right)$$

Case Study: Southwest Airlines

- Market capitalization \$11.1b, Rating BBB
- Time series of 5-year CDS spread/0.6 (solid line), physical default intensity $\lambda_{it}^{\mathbb{P}}$ (dashed line), risk-neutral default intensity $\lambda_{it}^{\mathbb{Q}}$ (dash-dotted line):



Risk Premia

- Jump-to-Default (JTD) risk premium: $\eta_{it}^{JTD} = \frac{\lambda_{it}^{\mathbb{Q}}}{\lambda_{it}^{\mathbb{P}}}$
- Under conditional diversification hypothesis (Jarrow, Lando, and Yu (2005)), JTD risk premium equal to one, since JTD risk can be diversified away
- Market price of (diffusive) risk for the firm-specific factors X_i and common factor Y :

$$\eta_{it}^{MTM}(X_{it}) = \frac{\kappa^{\mathbb{Q}}\theta_i^{\mathbb{Q}} - \kappa^{\mathbb{P}}\theta_i^{\mathbb{P}}}{\sigma_i\sqrt{X_{it}}} + \frac{\kappa^{\mathbb{Q}} - \kappa^{\mathbb{P}}}{\sigma_i}\sqrt{X_{it}}$$

$$\eta_{t}^{MTM}(Y_{t}) = \frac{\kappa^{\mathbb{Q}}\theta_Y^{\mathbb{Q}} - \kappa^{\mathbb{P}}\theta_Y^{\mathbb{P}}}{\sigma_Y\sqrt{Y_{t}}} + \frac{\kappa^{\mathbb{Q}} - \kappa^{\mathbb{P}}}{\sigma_Y}\sqrt{Y_{t}}$$

- Jump risk premium:

$$\eta_t^J(Y) = \frac{l_Y^{\mathbb{Q}}\mu_Y^{\mathbb{Q}} - l_Y^{\mathbb{P}}\mu_Y^{\mathbb{P}}}{l_Y^{\mathbb{P}}\mu_Y^{\mathbb{P}}}$$

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Research in Progress and Open Questions

Ongoing Research:

- Estimating the joint model for $\lambda_i^{\mathbb{P}}$ and $\lambda_i^{\mathbb{Q}}$
- Decompose tranche spreads into different components: pure default risk, liquidity component, various risk premia

How to:

- incorporate more than one common factor driving co-movements in default intensities
- incorporate correlation between default intensities and recovery rates
- price credit options and forward-starting CDOs in this framework

Summary

- Affine Jump Diffusion models
  - Allow the pricing of a large class of credit derivative securities via Fourier transform methods, without Monte-Carlo simulation
  - Just as fast as Copula model, since recursive ASB-step is bottleneck
- Model Fit:
  - Fit of term-structure of tranche spreads reasonably well, except for 3%-7% tranche
  - Size of tranche pricing errors might be proxy for market efficiency
- Risk Premia:
  - Jump-to-Default risk seems to be priced, i.e. $\eta_{it}^{JTD} > 1$
  - Work remains: analyze other types of risk premia

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